Find the Fourier series of the given function $f(x)$, which is assumed to have the period 2π, where $f(x)=0$ if $-\pi<x<0$, $f(x)=1$ if $0<x<\pi$. [100 嘉大資工 5]

[解] $f(x)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$,

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}\int_0^{\pi}1\cdot dx=1$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx=\frac{1}{\pi}\int_0^{\pi}1\cdot\cos nxdx=\frac{1}{n\pi}\cdot\sin nx\Big|_0^{\pi}=0$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx=\frac{1}{\pi}\int_0^{\pi}1\cdot\sin nxdx=-\frac{1}{n\pi}\cdot\cos nx\Big|_0^{\pi}=-\frac{1}{n\pi}(\cos n\pi-1)$$

$$=-\frac{1}{n\pi}[(-1)^n-1]=\begin{cases}0, & n\text{為偶數}\\ \dfrac{2}{n\pi}, & n\text{為奇數}\end{cases}=\frac{2}{(2n-1)\pi}$$

$$f(x)=\frac{1}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)x$$