A force field $\mathbf{F}$ in 3-space is given $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + (xz - y)\mathbf{k}$. Compute the work done by this force in moving a particle from (0, 0, 0) to (1, 2, 4) along the line segment joining these two points. [103 北科大化工 6]

[解]連接兩點的直線方程式為$\frac{x}{1} = \frac{y}{2} = \frac{z}{4} \Rightarrow$ 令 $x = t,\ y = 2t,\ z = 4t$

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + (xz - y)\mathbf{k} = t\mathbf{i} + 2t\mathbf{j} + (t \cdot 4t - 2t)\mathbf{k} = t\mathbf{i} + 2t\mathbf{j} + (4t^2 - 2t)\mathbf{k}$$

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Rightarrow d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k} = dt\mathbf{i} + 2dt\mathbf{j} + 4dt\mathbf{k} = (\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})dt$$

作功為$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C [t\mathbf{i} + 2t\mathbf{j} + (4t^2 - 2t)\mathbf{k}] \cdot (\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})dt = \int_0^1 [t + 4t + 4(4t^2 - 2t)]dt$

$$= \int_0^1 (16t^2 - 3t)dt = \frac{16}{3}t^3 - \frac{3}{2}t^2 \Big|_0^1 = \frac{16}{3} - \frac{3}{2} = \frac{23}{6}$$