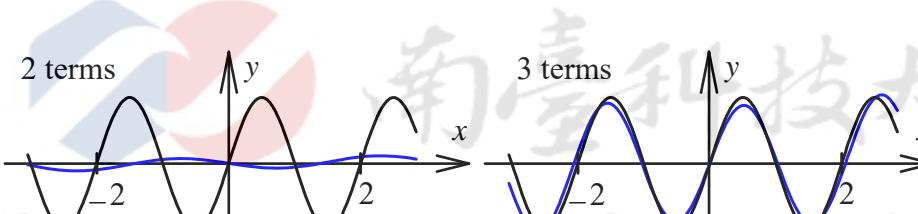


If $f(t) = \sin \pi t$ for $t \in (-\pi, \pi]$ be a function of period 2π . Find the Fourier Series representation of $f(t)$. [98 台灣聯大 C]

[解] $f(t)$ 為奇函數 \Rightarrow 設 $f(t) = \sum_{n=1}^{\infty} b_n \sin nt$

$$\begin{aligned}
 b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt dt = \frac{2}{\pi} \int_0^{\pi} \sin \pi t \sin nt dt = \frac{1}{\pi} \int_0^{\pi} [\cos(n-\pi)t - \cos(n+\pi)t] dt \\
 &= \frac{1}{\pi} \left[\frac{\sin(n-\pi)t}{n-\pi} - \frac{\sin(n+\pi)t}{n+\pi} \right]_0^{\pi} = \frac{1}{\pi} \left[\frac{\sin(n-\pi)\pi}{n-\pi} - \frac{\sin(n+\pi)\pi}{n+\pi} \right] \\
 &= \frac{1}{\pi} \left[\frac{\sin n\pi \cos \pi^2 - \cos n\pi \sin \pi^2}{n-\pi} - \frac{\sin n\pi \cos \pi^2 + \cos n\pi \sin \pi^2}{n+\pi} \right] \\
 &= \frac{1}{\pi} \left[\frac{0 \cdot \cos \pi^2 - (-1)^n \sin \pi^2}{n-\pi} - \frac{0 \cdot \cos \pi^2 + (-1)^n \sin \pi^2}{n+\pi} \right] = -\frac{(-1)^n \sin \pi^2}{\pi} \left(\frac{1}{n+\pi} + \frac{1}{n-\pi} \right) \\
 &= -\frac{(-1)^n \sin \pi^2}{\pi} \left[\frac{(n-\pi) + (n+\pi)}{n^2 - \pi^2} \right] = -\frac{2n(-1)^n \sin \pi^2}{\pi(n^2 - \pi^2)}
 \end{aligned}$$

$$f(t) = \sum_{n=1}^{\infty} -\frac{2n(-1)^n \sin \pi^2}{\pi(n^2 - \pi^2)} \sin nt = -\frac{2 \sin \pi^2}{\pi} \sum_{n=1}^{\infty} \frac{n(-1)^n}{n^2 - \pi^2} \sin nt$$



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