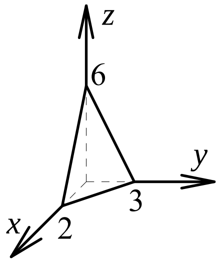
Let $\mathbf{F}=z\mathbf{j}+z\mathbf{k}$ represent the flow of a liquid. Find the flux of $\mathbf{F}$ through the surface S given by that portion of the plane $3x+2y+z=6$ in the first octant oriented upward. [104 彰師大物理甲光電甲 6]



[解]令 $f=3x+2y+z$，S的單位法向量為

$$\mathbf{n}=\frac{\nabla f}{|\nabla f|}=\frac{3\mathbf{i}+2\mathbf{j}+\mathbf{k}}{\sqrt{3^2+2^2+1^2}}=\frac{3\mathbf{i}+2\mathbf{j}+\mathbf{k}}{\sqrt{14}}$$

$$\iint_S \mathbf{F}\cdot d\mathbf{s}=\iint_S \mathbf{F}\cdot\mathbf{n}\frac{dxdy}{|\mathbf{n}\cdot\mathbf{k}|}=\iint_S (z\mathbf{j}+z\mathbf{k})\cdot\frac{3\mathbf{i}+2\mathbf{j}+\mathbf{k}}{\sqrt{14}}\frac{dxdy}{\left|\frac{3\mathbf{i}+2\mathbf{j}+\mathbf{k}}{\sqrt{14}}\cdot\mathbf{k}\right|}$$

$$=\iint_S 3zdxdy=\int_0^2\int_0^{3-3x/2}(18-9x-6y)dydx$$

$$=\int_0^2(18y-9xy-3y^2)\Big|_0^{3-3x/2}dx$$

$$=\int_0^2[(54-27x)-(27x-27x^2/2)-(27-27x+27x^2/4)]dx$$

$$=\int_0^2(27x^2/4-27x+27)dx=(9x^3/4-27x^2/2+27x)\Big|_0^2=18$$