Find the work ($\int_C \mathbf{F}\cdot d\mathbf{r}$) done by the force $\mathbf{F}(x, y, z) = e^x\mathbf{i} + xe^{xy}\mathbf{j} + xye^{xyz}\mathbf{k}$ acting along the smooth curve $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \le t \le 1$. [104 中興機械 5]

[解] $x = t,\ y = t^2, z = t^3 \Rightarrow \mathbf{F} = e^t\mathbf{i} + te^{t^3}\mathbf{j} + t^3e^{t^6}\mathbf{k},\ d\mathbf{r} = (\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k})dt,\ 0 \le t \le 1$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^1 (e^t\mathbf{i} + te^{t^3}\mathbf{j} + t^3e^{t^6}\mathbf{k})\cdot(\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k})dt = \int_0^1 (e^t + 2t^2e^{t^3} + 3t^5e^{t^6})dt$$

$$= (e^t + \frac{2}{3}e^{t^3} + \frac{3}{6}e^{t^6})\Big|_0^1 = (e-1) + \frac{2}{3}(e-1) + \frac{3}{6}(e-1) = \frac{13}{6}(e-1)$$