

Solve the differential equation $(x+3)^2y'' - 8(x+3)y' + 14y = 0$. [104 中興機械 1]

[解] 令 $x+3 = z \Rightarrow \frac{d}{dx} = \frac{d}{dz}$

原式 $\Rightarrow z^2 \frac{d^2y}{dz^2} - 8z \frac{dy}{dz} + 14y = 0 \dots\dots\dots$ (i)

令 $z = e^t \Rightarrow t = \ln z = \ln(x+3)$

(i) 式 $\Rightarrow \frac{d^2y}{dt^2} - 9 \frac{dy}{dt} + 14y = 0 \dots\dots\dots$ (ii)

(ii)式的特徵方程式 $\lambda^2 - 9\lambda + 14 = 0 \Rightarrow (\lambda - 2)(\lambda - 7) = 0 \Rightarrow \lambda = 2, 7$

$y = C_1 e^{2t} + C_2 e^{7t} = C_1(x+3)^2 + C_2(x+3)^7$

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