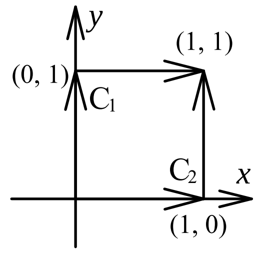
(1)Use the path (C_1) and (2)Use the path (C_2) in Fig. to evaluate the integral $\int_{\mathrm{O}}^{\mathrm{P}} r^2 d\mathbf{r}$, where $r^2 = x^2 + y^2$. [102 勤益電子 7]



[解](1)$(0, 0)$到$(0, 1): x = 0, dx = 0, r^2 = y^2, d\mathbf{r} = dy\mathbf{j}$

$$\int_{\mathrm{O}}^{\mathrm{A}} r^2 d\mathbf{r} = \int_0^1 y^2 dy\mathbf{j} = \left.\frac{y^3}{3}\right|_0^1 \mathbf{j} = \frac{1}{3}\mathbf{j}$$

$(0, 1)$到$(1, 1): y = 1, dy = 0, r^2 = x^2 + 1, d\mathbf{r} = dx\mathbf{i}$

$$\int_{\mathrm{A}}^{\mathrm{P}} r^2 d\mathbf{r} = \int_0^1 (x^2 + 1) dx\mathbf{i} = \left.(\frac{x^3}{3} + x)\right|_0^1 \mathbf{i} = \frac{4}{3}\mathbf{i}$$

$$\int_{\mathrm{O}}^{\mathrm{P}} r^2 d\mathbf{r} = \int_{\mathrm{O}}^{\mathrm{A}} r^2 d\mathbf{r} + \int_{\mathrm{A}}^{\mathrm{P}} r^2 d\mathbf{r} = \frac{1}{3}\mathbf{j} + \frac{4}{3}\mathbf{i}$$

(2)$(0, 0)$到$(1, 0): y = 0, dy = 0, r^2 = x^2, d\mathbf{r} = dx\mathbf{i}$

$$\int_{\mathrm{O}}^{\mathrm{B}} r^2 d\mathbf{r} = \int_0^1 x^2 dx\mathbf{i} = \left.\frac{x^3}{3}\right|_0^1 \mathbf{i} = \frac{1}{3}\mathbf{i}$$

$(1, 0)$到$(1, 1): x = 1, dx = 0, r^2 = 1 + y^2, d\mathbf{r} = dy\mathbf{j}$

$$\int_{\mathrm{B}}^{\mathrm{P}} r^2 d\mathbf{r} = \int_0^1 (1 + y^2) dy\mathbf{j} = \left.(y + \frac{y^3}{3})\right|_0^1 \mathbf{j} = \frac{4}{3}\mathbf{j}$$

$$\int_{\mathrm{O}}^{\mathrm{P}} r^2 d\mathbf{r} = \int_{\mathrm{O}}^{\mathrm{B}} r^2 d\mathbf{r} + \int_{\mathrm{B}}^{\mathrm{P}} r^2 d\mathbf{r} = \frac{1}{3}\mathbf{i} + \frac{4}{3}\mathbf{j}$$