

Compute the following values, where $i = \sqrt{-1}$:
(a) $(1+i)^{20}$ (b) $(\cos\theta + i\sin\theta)^6$, where $\theta = 30^\circ$
(c) $\ln(1+i)$ (d) i^i . [93 清大動機 5]

[解] (a) $(1+i)^{20} = [(1+i)^2]^{10} = (2i)^{10} = -1024$

(b) $(\cos 30^\circ + i \sin 30^\circ)^6 = \cos 180^\circ + i \sin 180^\circ = -1$

(c) $\ln(1+i) = \ln(\sqrt{2}e^{i(\frac{\pi}{4}+2k\pi)}) = \ln \sqrt{2} + \ln e^{i(\frac{\pi}{4}+2k\pi)} = \ln \sqrt{2} + i(\frac{\pi}{4} + 2k\pi), k = 0, \pm 1, \pm 2$

(d) $i^i = \exp(i \ln i) = \exp[i \ln e^{i(\frac{\pi}{2}+2k\pi)}] = \exp[i \cdot i(\frac{\pi}{2} + 2k\pi)] = e^{-\frac{\pi}{2}+2k\pi}, k = 0, \pm 1, \pm 2$

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