

Find the solution of the equation $(1+x^2)(dy-dx)=2xydx$ for which $y=1$ when $x=0$. [99 高師大電子 1]

[解]原式 $\Rightarrow dy-dx=\frac{2xy}{1+x^2}dx\Rightarrow y'-1=\frac{2xy}{1+x^2}\Rightarrow y'-\frac{2x}{1+x^2}y=1$

$$F=e^{\int \frac{2x}{1+x^2}dx}=e^{-\ln(1+x^2)}=\frac{1}{1+x^2}$$

$$y=\frac{1}{F}\left[\int F \cdot 1 dx + C\right]=(1+x^2)\left[\int \frac{1}{1+x^2} \cdot 1 dx + C\right]=(1+x^2)[\tan^{-1} x + C]$$

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