Let C be a circle $|z|=2$ described in the counterclockwise direction. (a)Compute the integral $\oint_C \frac{e^{kz^n}}{z}dz$, n is a positive integer. (b)Suppose the answer you obtained in part (a) is $in\pi$. Use part (a) to evaluate $\int_0^{2\pi} e^{2k\cos(n\theta)}\sin(2k\sin(n\theta))d\theta$. [104 中山電機甲丁己 5]

[解](a)在C內只有單極點 $z=0$

$$R_0 = e^{kz^n}\Big|_{z=0} = 1 \Rightarrow \oint_C \frac{e^{kz^n}}{z}dz = 2\pi i\cdot(1) = 2\pi i$$

(b)由(a)知 $n=2$

在C上 $z=2e^{i\theta} \Rightarrow dz = 2ie^{i\theta}d\theta$，$\frac{e^{kz^2}}{z}dz = \frac{e^{k\cdot 2^2 e^{i2\theta}}}{2e^{i\theta}}\cdot 2ie^{i\theta}d\theta = ie^{4ke^{i2\theta}}d\theta$

$$\oint_C \frac{e^{kz^2}}{z}dz = \int_0^{2\pi} ie^{4ke^{i2\theta}}d\theta = i\int_0^{2\pi} e^{4k(\cos 2\theta + i\sin 2\theta)}d\theta = i\int_0^{2\pi} e^{4k\cos 2\theta}\cdot e^{i4k\sin 2\theta}d\theta$$

$$= i\int_0^{2\pi} e^{4k\cos 2\theta}\{\cos[4k\sin(2\theta)] + i\sin[4k\sin(2\theta)]\}d\theta$$

$$= i\int_0^{2\pi} e^{4k\cos 2\theta}\cos[4k\sin(2\theta)]d\theta - \int_0^{2\pi} e^{4k\cos 2\theta}\sin[4k\sin(2\theta)]d\theta = 2\pi i$$

$$\int_0^{2\pi} e^{4k\cos 2\theta}\cos[4k\sin(2\theta)]d\theta = 2\pi,\quad \int_0^{2\pi} e^{4k\cos 2\theta}\sin[4k\sin(2\theta)]d\theta = 0$$