Please find the work required to move a particle by force $\mathbf{F}(x^2\mathbf{i}-yz\mathbf{j}+x\cos z\mathbf{k})$ along the path C($x=t^2, y=t, z=\pi t, 0\le t\le 3$) from point Q(9, 3, 3π) to point P(0, 0, 0) by calculate the integral $-\int_C \mathbf{F}\cdot d\mathbf{R}=-\int_C F_x dx+F_y dy+F_z dz$.[104 中央機械甲乙丙丁能源光機電乙 6;光機電甲 7]

[解] $x=t^2,\ y=t, z=\pi t \Rightarrow dx=2tdt,\ dy=dt,\ dz=\pi dt$

$\mathbf{F}=(t^2)^2\mathbf{i}-t\cdot\pi t\mathbf{j}+t^2\cos\pi t\mathbf{k}=t^4\mathbf{i}-\pi t^2\mathbf{j}+t^2\cos\pi t\mathbf{k},\ d\mathbf{r}=(2t\mathbf{i}+\mathbf{j}+\pi\mathbf{k})dt,\ 0\le t\le 3$

$$\int_C \mathbf{F}\cdot d\mathbf{r}=\int_3^0 (t^4\mathbf{i}-\pi t^2\mathbf{j}+t^2\cos\pi t\mathbf{k})\cdot(2t\mathbf{i}+\mathbf{j}+\pi\mathbf{k})dt=\int_3^0 (2t^5-\pi t^2+\pi t^2\cos\pi t)dt$$

$$=\left(\frac{2}{6}t^6-\frac{\pi}{3}t^3+t^2\sin\pi t\right)\Big|_3^0-\int_3^0 2t\sin\pi t dt=(-243+9\pi)+\frac{2}{\pi}\left(t\cos\pi t\Big|_3^0-\int_3^0\cos\pi t dt\right)$$

$$=(-243+9\pi)+\frac{2}{\pi}\left(3-\frac{\sin\pi t}{\pi}\Big|_3^0\right)=(-243+9\pi)+\frac{2}{\pi}\cdot 3=9\pi+\frac{6}{\pi}-243$$