

Find the Fourier series of the function $f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \end{cases}$. [91 成大造船 2]

[解] 設 $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$, 其中

$$a_0 = \int_0^2 f(x) dx = \int_0^1 x dx + \int_1^2 0 dx = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}$$

$$\begin{aligned} a_n &= \int_0^2 f(x) \cos n\pi x dx = \int_0^1 x \cos n\pi x dx = \frac{1}{n\pi} (x \sin n\pi x \Big|_0^1 - \int_0^1 \sin n\pi x dx) \\ &= \frac{\cos n\pi x}{n^2 \pi^2} \Big|_0^1 = \frac{\cos n\pi - 1}{n^2 \pi^2} = \frac{(-1)^n - 1}{n^2 \pi^2} \end{aligned}$$

$$\begin{aligned} b_n &= \int_0^2 f(x) \sin n\pi x dx = \int_0^1 x \sin n\pi x dx = -\frac{1}{n\pi} (x \cos n\pi x \Big|_0^1 - \int_0^1 \cos n\pi x dx) \\ &= -\frac{\cos n\pi}{n\pi} = -\frac{(-1)^n}{n\pi} \end{aligned}$$

$$f(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \left[\frac{(-1)^n - 1}{n^2 \pi^2} \cos n\pi x - \frac{(-1)^n}{n\pi} \sin n\pi x \right], \quad 0 \leq x < 2$$