Determine the solution $\bar{T}(x,s)$ in the Laplace transform domaim ($\bar{T}(x,s)$ is thw Laplace transform of $T(x, t)$) of the following initial-boundary value problem by Laplace transform technique:

$$\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial x^2},\ \text{for } 0<x<L,\ t>0$$

$$T(x,0)=T_0=\text{constant, for } 0<x<L$$

$$T(L,t)=\frac{\partial T}{\partial x}(0,t)=0,\ \text{for } t>0$$ [106成大機械6]

[解]將方程式左右兩邊對 t 取Laplace轉換

$$\int_0^\infty \frac{\partial T}{\partial t}e^{-st}dt=\int_0^\infty k\frac{\partial^2 T}{\partial x^2}e^{-st}dt \Rightarrow s\bar{T}-T(x,0)=k\bar{T}_{xx} \Rightarrow s\bar{T}-T_0=k\bar{T}_{xx}$$

$$\bar{T}_{xx}-\frac{s}{k}\bar{T}=-\frac{T_0}{k} \Rightarrow \bar{T}=Ae^{\sqrt{\frac{s}{k}}x}+Be^{-\sqrt{\frac{s}{k}}x}+\frac{T_0}{s}\cdots\cdots\cdots(\text{i})$$

$$\bar{T}_x=A\sqrt{\frac{s}{k}}e^{\sqrt{\frac{s}{k}}x}-B\sqrt{\frac{s}{k}}e^{-\sqrt{\frac{s}{k}}x}\cdots\cdots\cdots(\text{ii})$$

邊界條件對 t 取Laplace轉換，得

$$\bar{T}(L,s)=0,\ \bar{T}_x(0,s)=0$$

代入轉換的邊界條件，得

$$Ae^{\sqrt{\frac{s}{k}}L}+Be^{-\sqrt{\frac{s}{k}}L}+\frac{T_0}{s}=0,\ A\sqrt{\frac{s}{k}}-B\sqrt{\frac{s}{k}}=0 \Rightarrow A=B=-\frac{T_0}{2s\cosh(\sqrt{\frac{s}{k}}L)}$$

$$\bar{T}=-\frac{T_0}{2s\cosh(\sqrt{\frac{s}{k}}L)}(e^{\sqrt{\frac{s}{k}}x}+e^{-\sqrt{\frac{s}{k}}x})+\frac{T_0}{s}=\frac{T_0}{s}[1-\frac{\cosh(\sqrt{\frac{s}{k}}x)}{\cosh(\sqrt{\frac{s}{k}}L)}]$$