

Find the general solution $x^3 \frac{dy}{dx} + x^2y = 2y^{-4/3}$. [106 中興電機乙丙丁光電 8(1)]

[解] 原式 $\Rightarrow y' + x^{-1}y = 2x^{-3}y^{-4/3} \Rightarrow y^{4/3}y' + x^{-1}y^{7/3} = 2x^{-3}$ (i)

令 $u = y^{7/3} \Rightarrow u' = 7/3y^{4/3}y'$, 代入(i)式

$3/7u' + x^{-1}u = 2x^{-3} \Rightarrow u' + 7x^{-1}u/3 = 14x^{-3}/3$ (i)

$$F = e^{\int 7x^{-1}/3 dx} = e^{7/3 \cdot \ln x} = x^{7/3}$$

$$u = \frac{1}{F} [\int F \cdot 14x^{-3}/3 dx + C] = \frac{1}{x^{7/3}} [\int x^{7/3} \cdot 14x^{-3}/3 dx + C]$$

$$= x^{-7/3} [14x^{1/3} + C] = 14x^{-2} + Cx^{-7/3}$$

$$y^{7/3} = 14x^{-2} + Cx^{-7/3}$$

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