Given $f(x) = L - x$, $0 < x < L$, represent $f(x)$ by a Fourier sine series. Hint: $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$, where $b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$, $n = 1, 2, \cdots$. [102 中原機械丙 6]

[解]設 $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx = \frac{2}{L}\int_0^L (L - x)\sin\frac{n\pi x}{L}\,dx$$

$$= -\frac{2}{n\pi}\left[L\cos\frac{n\pi x}{L}\Big|_0^L - \left(x\cos\frac{n\pi x}{L}\Big|_0^L - \int_0^L \cos\frac{n\pi x}{L}\,dx\right)\right]$$

$$= -\frac{2}{n\pi}[L(\cos n\pi - 1) - (L\cos n\pi - 0)] = \frac{2}{n\pi}$$

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{L}, \quad 0 < x < L$$