Let $\mathbf{A}=\begin{bmatrix} 2 & 2 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, find a symmetric matrix $\mathbf{B}$ and a skew symmetric matrix $\mathbf{C}$, such that $\mathbf{B}+\mathbf{C}$

$=\mathbf{A}$. [94 中央機械 6]

[解]$\mathbf{A}=\begin{bmatrix} 2 & 2 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \mathbf{A}^{\mathrm{T}}=\begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ -1 & 0 & 0 \end{bmatrix}$

$$\mathbf{A}=\frac{\mathbf{A}+\mathbf{A}}{2}+\frac{\mathbf{A}^{\mathrm{T}}-\mathbf{A}^{\mathrm{T}}}{2}=\frac{\mathbf{A}+\mathbf{A}^{\mathrm{T}}}{2}+\frac{\mathbf{A}-\mathbf{A}^{\mathrm{T}}}{2}$$

$$\mathbf{B}=\frac{\mathbf{A}+\mathbf{A}^{\mathrm{T}}}{2}=\begin{bmatrix} 2 & 3/2 & -1/2 \\ 3/2 & -1 & 1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix},\ \mathbf{C}=\frac{\mathbf{A}-\mathbf{A}^{\mathrm{T}}}{2}=\begin{bmatrix} 0 & 1/2 & -1/2 \\ -1/2 & 0 & -1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$