

Find the general solution to the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^3 y^3$. [106 台大海洋丙四]

[解] 原式 $\Rightarrow y^{-3} y' + \frac{1}{x} y^{-2} = x^3 \dots \dots \dots \text{(i)}$

$$\Leftrightarrow u = y^{-2} \Rightarrow u' = -2y^{-3}y'$$

$$\text{(i)} \Rightarrow -\frac{1}{2}u' + \frac{1}{x}u = x^3 \Rightarrow u' - \frac{2}{x}u = -2x^3$$

$$F = e^{\int \frac{-2}{x} dx} = e^{-2\ln x} = x^{-2}$$

$$u = \frac{1}{x^{-2}} \left[\int x^{-2} \cdot (-2x^3) dx + C \right] = x^2 [(-x^2) + C] = -x^4 + Cx^2$$

$$y^{-2} = -x^4 + Cx^2$$

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