已知 $f(x)=x,\ 0<x<2$ 表為傅立葉正弦級數為 $x=\sum_{n=1}^{\infty}\frac{-4}{n\pi}\cos(n\pi)\sin(\frac{n\pi x}{2})$，試利用積分求 $F(x)=x^2,\ 0<x<2$ 之傅立葉級數。[105 屏科大車輛 8]

[解]將 $f(x)$的Fourier級數積分得

$$\frac{x^2}{2}=\sum_{n=1}^{\infty}\frac{-4}{n\pi}\cos(n\pi)[-\frac{2}{n\pi}\cos(\frac{n\pi x}{2})]+k=\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^2}\cos(\frac{n\pi x}{2})+k$$

$$x^2=\frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^2}\cos(\frac{n\pi x}{2})+C$$

$x=0$ 代入 $\Rightarrow 0=\frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^2}+C\cdots\cdots\cdots(\mathrm{i})$

而$\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^2}=-\frac{1}{1^2}+\frac{1}{2^2}-\frac{1}{3^2}+\cdots=-\frac{\pi^2}{12}$

$(\mathrm{i})\Rightarrow 0=\frac{16}{\pi^2}\cdot(-\frac{\pi^2}{12})+C\Rightarrow C=\frac{4}{3}$

$$\therefore x^2=\frac{4}{3}+\frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^2}\cos(\frac{n\pi x}{2}),\quad 0<x<2$$