

Is the integration $\int (3x^2 + 3y - 1)dx + (z^2 + 3x)dy + (2yz + 1)dz$ independent of the path? [98 屏教大光電 1]

[解] $\mathbf{F} = (3x^2 + 3y - 1)\mathbf{i} + (z^2 + 3x)\mathbf{j} + (2yz + 1)\mathbf{k} \Rightarrow F_1 = 3x^2 + 3y - 1, F_2 = z^2 + 3x, F_3 = 2yz + 1$

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2 + 3y - 1 & z^2 + 3x & 2yz + 1 \end{vmatrix} = 0 \Rightarrow \text{積分值與路徑無關}$$