Find the line integral $\int_C zdx + xdy + ydz$, where C is the triangle with vertices (3, 0, 0), (0, 0, 2), (0, 6, 0) traversed in the given order. [99 宜蘭電機 6]

[解]由$(3, 0, 0)$到$(0, 0, 2)$: $\begin{cases} y = 0 \\ \dfrac{x}{3} = \dfrac{z-2}{-2} \end{cases} \Rightarrow x = 3t,\ y = 0, z = -2t + 2 \Rightarrow dx = 3dt,\ dy = 0,\ dz = -2dt$

$\int_{C_1} zdx + xdy + ydz = \int_1^0 [(-2t+2)(3dt) + 0 + 0] = (-3t^2 + 6t)\Big|_1^0 = 3 - 6 = -3$

由$(0, 0, 2)$到$(0, 6, 0)$: $\begin{cases} x = 0 \\ \dfrac{y}{6} = \dfrac{z-2}{-2} \end{cases} \Rightarrow x = 0,\ y = 6t, z = -2t + 2 \Rightarrow dx = 0,\ dy = 6dt,\ dz = -2dt$

$\int_{C_2} zdx + xdy + ydz = \int_0^1 [0 + 0 + 6t(-2dt) + 0 + 0] = -6t^2\Big|_0^1 = -6$

由$(0, 6, 0)$到$(3, 0, 0)$: $\begin{cases} z = 0 \\ \dfrac{x-3}{-3} = \dfrac{y}{6} \end{cases} \Rightarrow x = -3t + 3,\ y = 6t, z = 0 \Rightarrow dx = -3dt,\ dy = 6dt,\ dz = 0$

$\int_{C_3} zdx + xdy + ydz = \int_1^0 [0 + (-3t+3)(6dt) + 0] = (-9t^2 + 18t)\Big|_1^0 = 9 - 18 = -9$

$$\int_C zdx + xdy + ydz = \int_{C_1} zdx + xdy + ydz + \int_{C_2} zdx + xdy + ydz + \int_{C_3} zdx + xdy + ydz$$
$$= -3 - 6 - 9 = -18$$