

Evaluate the surface integral $\oiint_S 4x dydz - z dx dy$, over the sphere $S: x^2 + y^2 + z^2 = 4$. [101 彰師大電機 5]

[解] 設 $\mathbf{F} = F_1 \mathbf{i} + F_2 \mathbf{j} + F_3 \mathbf{k}$, 則 $\mathbf{F}$ 對 S 的面積分為 $\iiint_S \mathbf{F} \cdot d\mathbf{s} = \iiint_S (F_1 dydz + F_2 dzdx + F_3 dxdy)$

$$\text{由原題知 } F_1 = 4x, F_2 = 0, F_3 = -z \Rightarrow \nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 4 + 0 - 1 = 3$$

$$\text{由散度定理知 } \oiint_S \mathbf{F} \cdot d\mathbf{s} = \iiint_V \nabla \cdot \mathbf{F} dv = \iiint_V 3 dv = 3V = 3 \cdot \frac{4}{3} \pi r^3 = 3 \cdot \frac{4}{3} \pi \cdot 2^3 = 32\pi$$