

Please find the unit normal vector of surface $xz^2 - 2xy - 6x = 8$ at point $P(1, -1, 2)$, and please find the tangent plane at that point. [100 嘉義土木與水資源 3]

[解] 令 $f(x, y, z) = xz^2 - 2xy - 6x$ ，則 $f=8$ 為該曲面，

$$\nabla f = (z^2 - 2y - 6)\mathbf{i} - 2x\mathbf{j} + 2xz\mathbf{k} \Rightarrow \nabla f|_{(1, -1, 2)} = -2\mathbf{j} + 4\mathbf{k}$$

在 P 點垂直該曲面的單位向量為

$$\mathbf{n} = \frac{\nabla f}{|\nabla f|} \Big|_{(1, -1, 2)} = \frac{-2\mathbf{j} + 4\mathbf{k}}{\sqrt{(-2)^2 + 4^2}} = -\frac{1}{\sqrt{5}}\mathbf{j} + \frac{2}{\sqrt{5}}\mathbf{k}$$

過 P 點的切平面為

$$(-2\mathbf{j} + 4\mathbf{k}) \cdot [(x-1)\mathbf{i} + (y+1)\mathbf{j} + (z-2)\mathbf{k}] = 0 \Rightarrow (y+1) + 2(z-2) = 0 \Rightarrow y - 2z + 5 = 0$$