

Find the complex Fourier series of the  $f(x)$  on the given interval. [104 清大生醫丙 8]

$$f(x) = \begin{cases} 0, & -\frac{1}{2} < x < -\frac{1}{4} \\ 1, & -\frac{1}{4} < x < \frac{1}{4} \\ 0, & \frac{1}{4} < x < \frac{1}{2} \end{cases}$$

[解] 令  $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i2n\pi x}, \quad -\frac{1}{2} < x < \frac{1}{2}$

$$c_n = 2 \int_{-1/2}^{1/2} f(x) e^{-i2n\pi x} dx = 2 \int_{-1/4}^{1/4} 1 \cdot e^{-i2n\pi x} dx = \frac{1}{-in\pi} e^{-i2n\pi x} \Big|_{-1/4}^{1/4}$$

$$= \frac{1}{-in\pi} (e^{-in\pi/2} - e^{in\pi/2}) = \frac{1}{in\pi} \cdot 2 \sinh \frac{n\pi}{2}$$

$$\therefore f(x) = \frac{2}{i\pi} \sum_{n=-\infty}^{\infty} \frac{1}{n} \sinh \frac{n\pi}{2} e^{i2n\pi x}, \quad -\frac{1}{2} < x < \frac{1}{2}$$