Find the surface integral of the vector function $\mathbf{F}=y^3\mathbf{i}+x^3\mathbf{j}+z^3\mathbf{k}$ over the portion of the surface defined as S: $x^2+4y^2=4,\quad x\geq 0,\quad y\geq 0,\quad 0\leq z\leq h$. [101 南大電機 9]

[解]Let $f=x^2+4y^2$，S的單位法向量為

$$\mathbf{n}=\frac{\nabla f}{|\nabla f|}=\frac{2x\mathbf{i}+8y\mathbf{j}}{\sqrt{(2x)^2+(8y)^2}}=\frac{2x\mathbf{i}+8y\mathbf{j}}{2\sqrt{x^2+16y^2}}=\frac{x\mathbf{i}+4y\mathbf{j}}{\sqrt{x^2+16y^2}}$$

$$\iint_S \mathbf{F}\cdot d\mathbf{s}=\iint_S \mathbf{F}\cdot\mathbf{n}\frac{dydz}{|\mathbf{n}\cdot\mathbf{i}|}=\iint_S (y^3\mathbf{i}+x^3\mathbf{j}+z^3\mathbf{k}/\cdot\frac{x\mathbf{i}+4y\mathbf{j}}{\sqrt{x^2+16y^2}}\frac{dydz}{\left|\frac{x\mathbf{i}+4y\mathbf{j}}{\sqrt{x^2+16y^2}}\cdot\mathbf{i}\right|}$$

$$=\iint_S \frac{xy^3+4x^3y}{|x|}dydz=\iint_S \frac{xy^3+4x^3y}{x}dydz=\iint_S (y^3+4x^2y)dydz$$

$$=\int_0^h\int_0^1 [y^3+4(4-4y^2)y]dydz=\int_0^h\int_0^1 (-15y^3+16y)dydz$$

$$=\int_0^h (-\frac{15}{4}y^4+8y^2)\Big|_0^1 dz=\int_0^h \frac{17}{4}dz=\frac{17}{4}h$$