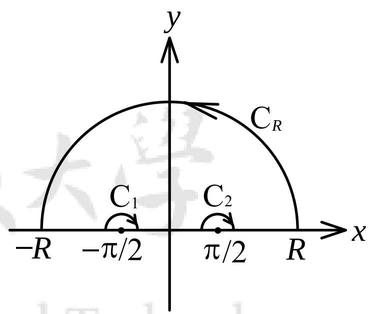
Please evaluate the integral $\int_{-\infty}^{\infty} \frac{\cos x}{\pi^2 - 4x^2} dx$. Please remember to draw your integral contour and show everything in detail. [89 成大機械 4]

[解]令 $f(z) = \frac{e^{iz}}{\pi^2 - 4z^2}$ 有單極點在 $z = -\pi/2, \pi/2$

$$R_{-\pi/2} = \left.\frac{e^{iz}}{-8z}\right|_{z=-\pi/2} = \frac{e^{i(-\pi/2)}}{4\pi} = -\frac{i}{4\pi}$$

$$R_{\pi/2} = \left.\frac{e^{iz}}{-8z}\right|_{z=\pi/2} = \frac{e^{i(\pi/2)}}{-4\pi} = -\frac{i}{4\pi}$$



$$\int_{C_R} \frac{e^{iz}}{\pi^2 - 4z^2} dz + \int_{-\infty}^{\infty} \frac{e^{ix}}{\pi^2 - 4x^2} dx - \pi i (R_{-\pi/2} + R_{\pi/2}) = 0$$

$$0 + \int_{-\infty}^{\infty} \frac{e^{ix}}{\pi^2 - 4x^2} dx - \pi i \left(-\frac{i}{4\pi} - \frac{i}{4\pi}\right) = 0$$

$$\int_{-\infty}^{\infty} \frac{e^{ix}}{\pi^2 - 4x^2} dx = \frac{1}{2} \Rightarrow \int_{-\infty}^{\infty} \frac{\cos x + i \sin x}{\pi^2 - 4x^2} dx = \frac{1}{2}$$

$$\int_{-\infty}^{\infty} \frac{\cos x}{\pi^2 - 4x^2} dx = \frac{1}{2}, \quad \int_{-\infty}^{\infty} \frac{\sin x}{\pi^2 - 4x^2} dx = 0$$