Expand $f(x)=x^2$ for $0<x<L$, in a cosine series. [100清大動機7(b)]

[解] $f(x)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\dfrac{n\pi x}{L}$

$$a_0=\frac{2}{L}\int_0^L f(x)dx=\frac{2}{L}\int_0^L x^2dx=\frac{2}{L}\cdot\frac{x^3}{3}\bigg|_0^L=\frac{2}{L}\cdot\frac{L^3}{3}=\frac{2L^2}{3}$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx=\frac{2}{L}\int_0^L x^2\cos\frac{n\pi x}{L}dx=\frac{2}{L}\cdot\frac{L}{n\pi}\left(x^2\sin\frac{n\pi x}{L}\bigg|_0^L-2\int_0^L x\sin\frac{n\pi x}{L}dx\right)$$

$$=\frac{2}{n\pi}\left[2\cdot\frac{L}{n\pi}\left(x\cos\frac{n\pi x}{L}\bigg|_0^L-\int_0^L\cos\frac{n\pi x}{L}dx\right)\right]=\frac{2}{n\pi}\left(\frac{2L}{n\pi}\cdot L\cos n\pi\right)=\frac{4L^2}{n^2\pi^2}(-1)^n$$

$$f(x)=\frac{L^2}{3}+\frac{4L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi x}{L}$$