

A curve is defined as $\mathbf{r}(t) = [a \cos t, a \sin t, ct]$, Please find $\mathbf{r}(s)$, where s is the arc length. [96 暨南
土木 4(a)]

[解] $\mathbf{v}(t) = -a \sin t \mathbf{i} + a \cos t \mathbf{j} + c \mathbf{k}$

$$\mathbf{v} \cdot \mathbf{v} = a^2 + c^2$$

$$s = \int_0^t \sqrt{a^2 + c^2} dt = t\sqrt{a^2 + c^2} \Rightarrow t = \frac{s}{\sqrt{a^2 + c^2}}$$

$$\mathbf{r}(s) = a \cos \frac{s}{\sqrt{a^2 + c^2}} \mathbf{i} + a \sin \frac{s}{\sqrt{a^2 + c^2}} \mathbf{j} + c \frac{s}{\sqrt{a^2 + c^2}} \mathbf{k}$$