

Solve the following ordinary differential equation [104 逢甲電子 1]

$$(a)y''(t) - 7y'(t) + 6y(t) = 2t \quad (b)y''(t) + 6y'(t) + 8y(t) = \sin 3t$$

[解](a)特徵方程式 $\lambda^2 - 7\lambda + 6 = 0 \Rightarrow (\lambda - 1)(\lambda - 6) = 0 \Rightarrow \lambda = 1, 6$

$$y_h = c_1 e^t + c_2 e^{6t}$$

$$\text{令 } y_p = At + B \Rightarrow y_p' = A \Rightarrow y_p'' = 0$$

代入原式

$$0 - 7A + 6(At + B) = 2t \Rightarrow A = \frac{1}{3}, B = \frac{7}{18}$$

$$\text{得 } y(t) = y_h + y_p = c_1 e^t + c_2 e^{6t} + \frac{1}{3}t + \frac{7}{18}$$

(b)特徵方程式 $\lambda^2 + 6\lambda + 8 = 0 \Rightarrow (\lambda + 2)(\lambda + 4) = 0 \Rightarrow \lambda = -2, -4$

$$y_h = c_1 e^{-2t} + c_2 e^{-4t}$$

$$\text{令 } y_p = A \sin 3t + B \cos 3t \Rightarrow y_p' = 3A \cos 3t - 3B \sin 3t \Rightarrow y_p'' = -9A \sin 3t - 9B \cos 3t$$

代入原式

$$(-9A \sin 3t - 9B \cos 3t) + 6(3A \cos 3t - 3B \sin 3t) + 8(A \sin 3t + B \cos 3t) = \sin 3t$$

$$(-A - 18B) \sin 3t + (18A - B) \cos 3t = \sin 3t$$

$$\begin{cases} -A - 18B = 1 \\ 18A - B = 0 \end{cases} \Rightarrow A = -\frac{1}{325}, B = -\frac{18}{325}$$

$$\text{得 } y(t) = y_h + y_p = c_1 e^{-2t} + c_2 e^{-4t} - \frac{1}{325} \sin 3t - \frac{18}{325} \cos 3t$$