Find the Fourier series of $f(x)$ as given over one period. $f(x)=x^2\ (-\frac{\pi}{2}<x<\frac{\pi}{2})$. [105成大造船2]

[解] $f(x)$為偶函數，週期為$\pi \Rightarrow$ 設 $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos 2nx$

$$a_0=\frac{4}{\pi}\int_0^{\frac{\pi}{2}} f(x)dx=\frac{4}{\pi}\int_0^{\frac{\pi}{2}} x^2 dx=\frac{4}{\pi}\cdot\frac{x^3}{3}\Big|_0^{\frac{\pi}{2}}=\frac{4}{\pi}\cdot\frac{\pi^3}{24}=\frac{\pi^2}{6}$$

$$a_n=\frac{4}{\pi}\int_0^{\frac{\pi}{2}} f(x)\cos 2nx dx=\frac{4}{\pi}\int_0^{\frac{\pi}{2}} x^2\cos 2nx dx=\frac{4}{\pi}\cdot\frac{1}{2n}(x^2\sin 2nx\Big|_0^{\frac{\pi}{2}}-2\int_0^{\frac{\pi}{2}} x\sin 2nx dx)$$

$$=\frac{2}{n\pi}[2\cdot\frac{1}{2n}(x\cos 2nx\Big|_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\cos 2nx dx)]=\frac{2}{n\pi}(\frac{1}{n}\cdot\frac{\pi}{2}\cos n\pi)=\frac{(-1)^n}{n^2}$$

$$f(x)=\frac{\pi^2}{12}+\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos 2nx$$