Show that the differential form under the integral sign of $I = \int_{(-1,5)}^{(4,3)} (3z^2 dx + 6xzdz)$ is exact, so that we have independence of path in any domain, and find the value of the integral I from A(−1, 5) to B(4, 3). [97 中央機械丁光電甲 4]

[解]令$F_1 = 3z^2,\ F_2 = 6xz \Rightarrow \dfrac{\partial F_1}{\partial z} = 6z,\ \dfrac{\partial F_2}{\partial x} = 6z$

$\dfrac{\partial F_1}{\partial z} = \dfrac{\partial F_2}{\partial x} \Rightarrow 3z^2dx + 6xzdz$為正合

設$\phi(x, z)$滿足$\dfrac{\partial \phi}{\partial x} = F_1,\ \dfrac{\partial \phi}{\partial z} = F_2$，則

$\phi = \int_x F_1 dx + f(z) = \int_x 3z^2 dx + f(z) = 3xz^2 + f(z)$

$\phi = \int_z F_2 dz + g(x) = \int_z 6xzdz + g(x) = 3xz^2 + g(x)$

比較兩式，得$\phi(x, z) = 3xz^2$

$I = \int_{(-1,5)}^{(4,3)} (3z^2dx + 6xzdz) = \phi(x, z)\Big|_{(-1,5)}^{(4,3)} = 3\cdot 4\cdot 3^2 - 3\cdot(-1)\cdot 5^2 = 183$