

Find the Fourier transform of the given function $f(x) = \begin{cases} xe^{-x}, & x > 0 \\ 0, & x < 0 \end{cases}$. [100 中原機械丙 5]

$$\begin{aligned}[解] \int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx &= \int_0^{\infty} xe^{-x} e^{-i\omega x} dx = \int_0^{\infty} xe^{-(1+i\omega)x} dx = -\frac{1}{1+i\omega} [xe^{-(1+i\omega)x}]_0^{\infty} - \int_0^{\infty} e^{-(1+i\omega)x} dx \\ &= -\frac{1}{1+i\omega} \left[0 + \frac{e^{-(1+i\omega)x}}{1+i\omega} \right]_0^{\infty} = -\frac{0-1}{(1+i\omega)^2} = \frac{1}{(1+i\omega)^2}\end{aligned}$$

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