Using the calculus of residues to show $\int_{-\infty}^{\infty} \frac{\cos x}{x^2+a^2}dx = \frac{\pi}{a}e^{-a}$, $a>0$. [97 海洋光電 10(b)]

[解]令 $f(z)=\frac{e^{iz}}{z^2+a^2}$，則$\int_{-\infty}^{\infty}\frac{\cos xdx}{x^2+a^2}$為$\int_{-\infty}^{\infty} f(z)dz$的實部

$$\mathrm{Res}[f(z);ai]=\left.\frac{e^{iz}}{2z}\right|_{z=ai}=\frac{e^{i(ai)}}{2ai}=-\frac{e^{-a}}{2a}i$$

$$\int_{-\infty}^{\infty}\frac{\cos xdx}{x^2+a^2}=\mathrm{Re}[2\pi i\cdot(-\frac{e^{-a}}{2a}i)]=\frac{\pi}{a}\cdot e^{-a}$$