

Find the general solution of $y'' + 2y' + y = 3e^{-x}$. [99 交大機械甲 3]

[解]特徵方程式為

$$\lambda^2 + 2\lambda + 1 = 0 \Rightarrow \lambda = -1, -1$$

$$y_h(x) = (C_1 + C_2x)e^{-x}$$

$$\text{Let } y_p(x) = Ax^2e^{-x} \Rightarrow y_p' = (-Ax^2 + 2Ax)e^{-x} \Rightarrow y_p'' = (Ax^2 - 4Ax + 2A)e^{-x}$$

代入原式得

$$(Ax^2 - 4Ax + 2A)e^{-x} + 2(-Ax^2 + 2Ax)e^{-x} + Ax^2e^{-x} = 3e^{-x} \Rightarrow A = \frac{3}{2}$$

$$\text{解為 } y(x) = y_h(x) + y_p(x) = \left(C_1 + C_2x + \frac{3}{2}x^2\right)e^{-x}$$

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