Express the values of the following complex numbers in the form of $a+ib$, where a, b are real.
(1)$(2i)^{3i}$ (2)$(1+i)^{1-i}$. [105 中正光電 7]

[解](1) $(2i)^{3i}=\exp[3i\ln(2i)]=\exp\{3i\ln[2e^{i(\frac{\pi}{2}+2k\pi)}]\}=\exp\{3i[\ln 2+i(\frac{\pi}{2}+2k\pi)]\}$

$$=\exp[-3(\frac{\pi}{2}+2k\pi)+i3\ln 2]=\exp[-3(\frac{\pi}{2}+2k\pi)]\cdot\exp(i3\ln 2)$$

$$=\exp[-3(\frac{\pi}{2}+2k\pi)][\cos(3\ln 2)+i\sin(3\ln 2)]$$

(2) $(1+i)^{1-i}=\exp[(1-i)\ln(1+i)]=\exp\{(1-i)\ln[\sqrt{2}e^{i(\frac{\pi}{4}+2k\pi)}]\}$

$$=\exp\{(1-i)[\ln\sqrt{2}+i(\frac{\pi}{4}+2k\pi)]\}$$

$$=\exp[(\ln\sqrt{2}+\frac{\pi}{4}+2k\pi)+i(\frac{\pi}{4}+2k\pi-\ln\sqrt{2})]$$

$$=\sqrt{2}\exp(\frac{\pi}{4}+2k\pi)\cdot\exp i(\frac{\pi}{4}+2k\pi-\ln\sqrt{2})=\sqrt{2}\exp(\frac{\pi}{4}+2k\pi)\cdot\exp i(\frac{\pi}{4}-\ln\sqrt{2})$$

$$=\sqrt{2}\exp(\frac{\pi}{4}+2k\pi)[\cos(\frac{\pi}{4}-\ln\sqrt{2})+i\sin(\frac{\pi}{4}-\ln\sqrt{2})]$$