Find the directional derivative of $F(x, y, z) = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction of $6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. [104 中原機械甲 6]

[解]$\nabla F = \dfrac{\partial F}{\partial x}\mathbf{i} + \dfrac{\partial F}{\partial y}\mathbf{j} + \dfrac{\partial F}{\partial z}\mathbf{k} = (y^2 - 8xy)\mathbf{i} + (2xy - 4x^2)\mathbf{j} + 2z\mathbf{k}$

$$\nabla F\big|_{(1,-1,2)} = [(-1)^2 - 8\cdot 1\cdot(-1)]\mathbf{i} + [2\cdot 1\cdot(-1) - 4\cdot 1^2]\mathbf{j} + 2\cdot 2\mathbf{k} = 9\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$$

令 $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \Rightarrow \mathbf{u}_a = \dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{6^2 + 2^2 + 3^2}} = \dfrac{6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{7}$

所求為$\nabla F\big|_{(1,-1,2)} \cdot \mathbf{u}_a = (9\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}) \cdot \dfrac{6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{7} = \dfrac{54 - 12 + 12}{7} = \dfrac{54}{7}$