Evaluate the integral with aid of residues: $\int_{-\infty}^{\infty} \frac{\cos x dx}{(x^2+a^2)(x^2+b^2)}$, where $a > b > 0$. [90 中正機械 5(a)]

[解]令 $f(z) = \frac{e^{iz}}{(z^2+a^2)(z^2+b^2)}$，則$\int_{-\infty}^{\infty} \frac{\cos x dx}{(x^2+a^2)(x^2+b^2)}$為$\int_{-\infty}^{\infty} f(z)dz$的實部

$$\mathrm{Res}[f(z); ai] = \left.\frac{e^{iz}}{2z(z^2+b^2)+(z^2+a^2)\cdot 2z}\right|_{z=ai} = \frac{e^{i(ai)}}{2ai(-a^2+b^2)} = \frac{e^{-a}}{2a(a^2-b^2)}i$$

$$\mathrm{Res}[f(z); bi] = \left.\frac{e^{iz}}{2z(z^2+b^2)+(z^2+a^2)\cdot 2z}\right|_{z=bi} = \frac{e^{i(bi)}}{(-b^2+a^2)\cdot 2bi} = -\frac{e^{-b}}{(a^2-b^2)\cdot 2b}i$$

$$\int_{-\infty}^{\infty} \frac{\cos x dx}{(x^2+a^2)(x^2+b^2)} = \mathrm{Re}[2\pi i \cdot (\frac{e^{-a}}{2a(a^2-b^2)}i - \frac{e^{-b}}{(a^2-b^2)\cdot 2b}i)] = -\frac{\pi}{a^2-b^2}\cdot(\frac{e^{-a}}{a} - \frac{e^{-b}}{b})$$