f(z) = u(x, y) + iv(x, y) is an analytic complex function. It is known that  $u(x, y) = x^3 + 3x^2y + axy^2 + by^3$ . Please find v(x, y) and the values of a and b. [92 中正機械 6]

[解]
$$v_y = u_x = 3x^2 + 6xy + ay^2 \Rightarrow v = 3x^2y + 3xy^2 + \frac{1}{3}ay^3 + g(x)$$

$$u_y = -v_x \Rightarrow 3x^2 + 2axy + 3by^2 = -[6xy + 3y^2 + g'(x)]$$
比較係數知  $a = -3, b = -1, g'(x) = -3x^2 \Rightarrow g(x) = -x^3$ 

$$v = 3x^2y + 3xy^2 - y^3 - x^3$$

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