

Find the integral $\int_{(2,0,1)}^{(4,4,0)} 2x(y^3 - z^3)dx + 3x^2y^2dy - 3x^2z^2dz$. [98 宜蘭生物機電 1]

[解]由(4, 4, 0)到(2, 0, 1): $\frac{x-2}{2} = \frac{y}{4} = \frac{z-1}{-1} \Rightarrow x = 2t + 2, y = 4t, z = -t + 1$

$$dx = 2dt, dy = 4dt, dz = -dt$$

$$\begin{aligned} & \int_{(2,0,1)}^{(4,4,0)} 2x(y^3 - z^3)dx + 3x^2y^2dy - 3x^2z^2dz \\ &= \int_{(2,0,1)}^{(4,4,0)} \{2(2t+2)[(4t)^3 - (-t+1)^3](2dt) + 3(2t+2)^2(4t)^2(4dt) - 3(2t+2)^2(-t+1)^2(-dt)\} \\ &= \int_1^0 \{8(t+1)[64t^3 - (-t^3 + 3t^2 - 3t + 1)] + 768t^2(t^2 + 2t + 1) + 12[(t+1)(-t+1)]^2\}dt \\ &= \int_1^0 [8(t+1)(65t^3 - 3t^2 + 3t - 1) + (768t^4 + 1536t^3 + 768t^2) + 12(1 - t^2)^2]dt \\ &= \int_1^0 [(520t^4 - 496t^3 + 16t - 8) + (768t^4 + 1536t^3 + 768t^2) + (12t^4 - 24t^2 + 12)]dt \\ &= \int_1^0 (1300t^4 + 1040t^3 + 744t^2 + 16t + 4)dt = (260t^5 + 260t^4 + 248t^3 + 8t^2 + 4t) \Big|_1^0 \\ &= -260 - 260 - 248 - 8 - 4 = -780 \end{aligned}$$