

某質點受外力  $\mathbf{F} = y\mathbf{i} + 2x\mathbf{j}$  作用沿以原點為圓心半徑為 1 之圓弧，自(1, 0)移動至(0, 1)，試計算其所做之功。[97 交大土木丁 13]

$$[\text{解}] \mathbf{F} = y\mathbf{i} + 2x\mathbf{j} = \sin t\mathbf{i} + 2\cos t\mathbf{j}, \quad \mathbf{r} = \cos t\mathbf{i} + \sin t\mathbf{j} \Rightarrow d\mathbf{r} = (-\sin t\mathbf{i} + \cos t\mathbf{j})dt, \quad 0 \leq t \leq \frac{\pi}{2}$$

$$\begin{aligned} \int_C \mathbf{F} \cdot d\mathbf{r} &= \int_0^{\frac{\pi}{2}} (\sin t\mathbf{i} + 2\cos t\mathbf{j}) \cdot (-\sin t\mathbf{i} + \cos t\mathbf{j})dt = \int_0^{\frac{\pi}{2}} (-\sin^2 t + 2\cos^2 t)dt \\ &= \int_0^{\frac{\pi}{2}} \left(-\frac{1 - \cos 2t}{2} + 2 \cdot \frac{1 + \cos 2t}{2}\right)dt = \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{3}{2}\cos 2t\right)dt \\ &= \left(\frac{t}{2} + \frac{3}{4}\sin 2t\right)\bigg|_0^{\frac{\pi}{2}} = \frac{\pi}{4} \end{aligned}$$