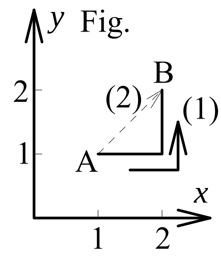
Calculate the line integral of function $\mathbf{v} = y^2 x\mathbf{i} + 2x(y+1)\mathbf{j}$ from the point A(1,1,0) to the point B(2, 2,0), along the path (1) and (2) in Fig. What is the line integral $\oint \mathbf{v} \cdot d\mathbf{l}$ for the loop that goes from A to B along (1) and return to A along (2)? [98 屏教大光電 10]



[解]along (1)

$$\int_{C_1} \mathbf{v} \cdot d\mathbf{l} = \int_{A}^{C} \mathbf{v} \cdot d\mathbf{l} + \int_{C}^{B} \mathbf{v} \cdot d\mathbf{l} = \int_1^2 [1^2 \cdot x\mathbf{i} + 2x(1+1)\mathbf{j}] \cdot dx\mathbf{i} + \int_1^2 [y^2 \cdot 2\mathbf{i} + 2 \cdot 2(y+1)\mathbf{j}] \cdot dy\mathbf{j}$$

$$= \int_1^2 x dx + \int_1^2 4(y+1) dy = \left.\frac{x^2}{2}\right|_1^2 + \left.(2y^2 + 4y)\right|_1^2 = \frac{3}{2} + (6+4) = \frac{23}{2}$$

along (2)

$$\int_{C_2} \mathbf{v} \cdot d\mathbf{l} = \int_1^2 [x^2 \cdot x\mathbf{i} + 2x(x+1)\mathbf{j}] \cdot (dx\mathbf{i} + dx\mathbf{j}) = \int_1^2 [x^3 + 2x(x+1)] dx$$

$$= \left.\left(\frac{x^4}{4} + \frac{2x^3}{3} + x^2\right)\right|_1^2 = \frac{15}{4} + \frac{14}{3} + 3 = \frac{137}{12}$$

$$\oint \mathbf{v} \cdot d\mathbf{l} = \int_{C_1} \mathbf{v} \cdot d\mathbf{l} - \int_{C_2} \mathbf{v} \cdot d\mathbf{l} = \frac{23}{2} - \frac{137}{12} = \frac{1}{12}$$