Find the characteristics of the following partial differential equation, reduce it to a standard form and then solve it: $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial x \partial y}-2\dfrac{\partial^2 u}{\partial y^2}=0$. [106 中興環工 7]

[解]$A=1,\ B=1,\ C=-2 \Rightarrow B^2-4AC=1+8=9$

$$\frac{dy}{dx}=\frac{B\pm\sqrt{B^2-4AC}}{2A}=\frac{1\pm 3}{2}=-1,\ 2$$

令$\xi=x+y,\ \eta=2x-y$

$u_x=u_\xi+2u_\eta,\ u_y=u_\xi-u_\eta$

$u_{xx}=(u_{\xi\xi}+2u_{\xi\eta})+2(u_{\eta\xi}+2u_{\eta\eta})=u_{\xi\xi}+4u_{\xi\eta}+4u_{\eta\eta}$

$u_{xy}=(u_{\xi\xi}-u_{\xi\eta})+2(u_{\eta\xi}-u_{\eta\eta})=u_{\xi\xi}+u_{\xi\eta}-2u_{\eta\eta}$

$u_{yy}=(u_{\xi\xi}-u_{\xi\eta})-(u_{\eta\xi}-u_{\eta\eta})=u_{\xi\xi}-2u_{\xi\eta}+u_{\eta\eta}$

原式$\Rightarrow (u_{\xi\xi}+4u_{\xi\eta}+4u_{\eta\eta})+(u_{\xi\xi}+u_{\xi\eta}-2u_{\eta\eta})-2(u_{\xi\xi}-2u_{\xi\eta}+u_{\eta\eta})=0 \Rightarrow 9u_{\xi\eta}=0$

解為$u=f(\xi)+g(\eta)=f(x+y)+g(2x-y)$