

Find the Fourier integral of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$. [97 宜蘭電子 3]

[解] $\hat{f}(x) = \int_0^\infty [a(\omega) \cos \omega x + b(\omega) \sin \omega x] d\omega$

$$a(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos \omega x dx = \frac{1}{\pi} \int_{-1}^1 \cos \omega x dx = \frac{1}{\pi \omega} [\sin(\omega) - \sin(-\omega)] = \frac{2 \sin \omega}{\pi \omega}$$

$$b(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin \omega x dx = 0$$

$$\hat{f}(x) = \frac{2}{\pi} \int_0^\infty \frac{\sin \omega}{\omega} \cos \omega x d\omega$$

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