Based on the Fourier integral, derive the explicit expression of $F(\omega)$ from the following equation:

$e^{-x^2} = \int_0^{\infty} F(\omega)\cos\omega x d\omega$. [103 中正化工 6]

[解] $$\int_{-\infty}^{\infty} e^{-x^2} e^{-i\omega x} dx = \int_{-\infty}^{\infty} e^{-(x^2+i\omega x)} dx = \int_{-\infty}^{\infty} e^{-(x+\frac{i\omega}{2})^2-\frac{\omega^2}{4}} dx = e^{-\frac{\omega^2}{4}} \int_{-\infty}^{\infty} e^{-(x+\frac{i\omega}{2})^2} dx$$

$$\text{令 } u = x + \frac{i\omega}{2} \Rightarrow \int_{-\infty}^{\infty} e^{-x^2} e^{-i\omega x} dx = e^{-\frac{\omega^2}{4}} \int_{-\infty}^{\infty} e^{-u^2} du = e^{-\frac{\omega^2}{4}} \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} e^{-x^2} (\cos\omega x - i\sin\omega x) dx = \sqrt{\pi} e^{-\frac{\omega^2}{4}} \Rightarrow \int_{-\infty}^{\infty} e^{-x^2} \cos\omega x dx = \sqrt{\pi} e^{-\frac{\omega^2}{4}}, \int_{-\infty}^{\infty} e^{-x^2} \sin\omega x dx = 0$$

$$\int_0^{\infty} e^{-x^2} \cos\omega x dx = \frac{1}{2}\int_{-\infty}^{\infty} e^{-x^2} \cos\omega x dx = \frac{\sqrt{\pi}}{2} e^{-\frac{\omega^2}{4}}$$

$$F(\omega) = \frac{2}{\pi}\int_0^{\infty} e^{-x^2} \cos\omega x dx = \frac{2}{\pi}\cdot\frac{\sqrt{\pi}}{2} e^{-\frac{\omega^2}{4}} = \frac{1}{\sqrt{\pi}} e^{-\frac{\omega^2}{4}}$$