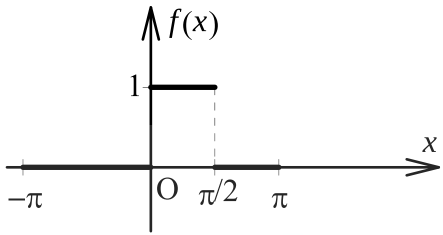
Find the Fourier series of the following function $f(x)$, which is assumed to have the period 2π. [104 高第一光電 7]



[解] $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{0}^{\pi/2} 1 \cdot dx = \frac{1}{\pi} \cdot x\Big|_0^{\pi/2} = \frac{1}{\pi} \cdot \frac{\pi}{2} = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx = \frac{1}{\pi}\int_{0}^{\pi/2} 1 \cdot \cos nx dx = \frac{1}{n\pi} \cdot \sin nx\Big|_0^{\pi/2} = \frac{1}{n\pi} \cdot \sin\frac{n\pi}{2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx = \frac{1}{\pi}\int_{0}^{\pi/2} 1 \cdot \sin nx dx = -\frac{1}{n\pi} \cdot \cos nx\Big|_0^{\pi/2} = -\frac{1}{n\pi}(\cos\frac{n\pi}{2} - 1)$$

$$f(x) = \frac{1}{4} + \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}[\sin\frac{n\pi}{2}\cos nx - (\cos\frac{n\pi}{2} - 1)\sin nx]$$