

Suppose a periodic function $f(t)$ with period is defined as $f(t) = \begin{cases} \frac{1}{k}, & 0 \leq t \leq k \\ 0, & k \leq t < 2 \end{cases}$, where k is a

constant ($0 < k < 2$). Please expand $f(t)$ in a Fourier series. [100中原機械甲6]

$$[\text{解}] f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi t + b_n \sin n\pi t)$$

$$a_0 = \int_0^2 f(t) dt = \int_0^k \frac{1}{k} dt = \left. \frac{t}{k} \right|_0^k = 1$$

$$a_n = \int_0^2 f(t) \cos n\pi t dt = \int_0^k \frac{1}{k} \cos n\pi t dt = \left. \frac{\sin n\pi t}{kn\pi} \right|_0^k = \frac{\sin kn\pi}{kn\pi}$$

$$b_n = \int_0^2 f(t) \sin n\pi t dt = \int_0^k \frac{1}{k} \sin n\pi t dt = - \left. \frac{\cos n\pi t}{kn\pi} \right|_0^k = - \frac{\cos kn\pi - 1}{kn\pi}$$

$$f(t) = \frac{1}{2} + \frac{1}{k\pi} \sum_{n=1}^{\infty} \left[\frac{\sin kn\pi}{n} \cos n\pi t - \frac{\cos kn\pi - 1}{n} \sin n\pi t \right]$$