

Solve the first order ordinary differential equation $\frac{dy}{dx} = \frac{x+y}{x-y}$. [106 台科大機械 1(a)]

[解]原式 $\Rightarrow (x+y)dx - (x-y)dy = 0 \cdots \cdots \cdots \text{(i)}$, 令 $y=ux \Rightarrow dy=xdu+udx$

$$\begin{aligned}\text{(i)} &\Rightarrow (x+ux)dx - (x-ux)(xdu+udx) = 0 \Rightarrow (1+u)dx - (1-u)(xdu+udx) = 0 \\ &[(1+u) - (1-u)udx] - (1-u)xdu = 0 \Rightarrow (u^2 + 1)udx - (1-u)xdu = 0\end{aligned}$$

$$\frac{dx}{x} + \frac{u-1}{u^2+1}du = 0 \Rightarrow \int \frac{dx}{x} + \int \left(\frac{u}{u^2+1} - \frac{1}{u^2+1} \right) du = k$$

$$\ln x + \frac{1}{2} \ln(u^2 + 1) - \tan^{-1} u = k \Rightarrow 2 \ln x + \ln(u^2 + 1) - 2 \tan^{-1} u = 2k$$

$$\ln[x^2(u^2 + 1)] - 2 \tan^{-1} u = C \Rightarrow \ln(y^2 + x^2) - 2 \tan^{-1} \frac{y}{x} = C$$

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