Solve the first order ordinary differential equation $\dfrac{dy}{dx}=\dfrac{x+y}{x-y}$. [106 台科大機械 1(a)]

[解]原式 $\Rightarrow (x+y)dx-(x-y)dy=0\cdots\cdots\cdots$(i)，令 $y=ux \Rightarrow dy=xdu+udx$

$$(\mathrm{i}) \Rightarrow (x+ux)dx-(x-ux)(xdu+udx)=0 \Rightarrow (1+u)dx-(1-u)(xdu+udx)=0$$

$$[(1+u)-(1-u)udx]-(1-u)xdu=0 \Rightarrow (u^2+1)udx-(1-u)xdu=0$$

$$\frac{dx}{x}+\frac{u-1}{u^2+1}du=0 \Rightarrow \int\frac{dx}{x}+\int\left(\frac{u}{u^2+1}-\frac{1}{u^2+1}\right)du=k$$

$$\ln x+\frac{1}{2}\ln(u^2+1)-\tan^{-1}u=k \Rightarrow 2\ln x+\ln(u^2+1)-2\tan^{-1}u=2k$$

$$\ln[x^2(u^2+1)]-2\tan^{-1}u=C \Rightarrow \ln(y^2+x^2)-2\tan^{-1}\frac{y}{x}=C$$