Use the method of Laplace transform to solve the initial value problem: $t\ddot{y}+(t-1)\dot{y}+y=0$, $y(0)=\dot{y}(0)=0$, $y(1)=1$. What will happen if $\dot{y}(0)\neq 0$? Why? [106 台科大機械 2]

[解]對原式取 Laplace轉換

$$-\frac{d}{ds}[s^2Y-sy(0)-\dot{y}(0)]-\frac{d}{ds}[sY-y(0)]-[sY-y(0)]+Y=0$$

$$-(s^2Y'+2sY)-[(sY'+Y)]-sY+Y=0$$

$$(-s^2-s)Y'-3sY=0\Rightarrow(s+1)Y'+3Y=0\Rightarrow\frac{dY}{Y}+\frac{3ds}{s+1}=0$$

$$\ln Y+3\ln(s+1)=k\Rightarrow Y(s+1)^3=C\Rightarrow Y=\frac{C}{(s+1)^3}\Rightarrow y=\frac{1}{2}Ct^2e^{-t}$$

$$y(1)=1\Rightarrow\frac{1}{2}Ce^{-1}=1\Rightarrow C=2e$$

$$y=t^2e^{1-t}\Rightarrow\dot{y}(t)=(2t-t^2)e^{1-t}\Rightarrow\dot{y}(0)=0$$

若 $\dot{y}(0)\neq 0$ 則不合

解為 $y(t)=t^2e^{1-t}$