

Evaluate $\int_{-\infty}^{\infty} \frac{\cos mx}{1+x^2} dx$. [105 成大機械 5]

[解]令 $f(z) = \frac{e^{imz}}{1+z^2}$, 則 $\int_{-\infty}^{\infty} \frac{\cos mx dx}{1+x^2}$ 為 $\int_{-\infty}^{\infty} f(z) dz$ 的實部

(1)當 $m > 0$ 時, 上半平面只有極點 i

$$\text{Res}[f(z); i] = \left. \frac{e^{imz}}{2z} \right|_{z=i} = \frac{e^{i(mi)}}{2i} = -\frac{e^{-m}}{2} i$$

$$\int_{-\infty}^{\infty} \frac{\cos mx dx}{1+x^2} = \text{Re}[2\pi i \cdot (-\frac{e^{-m}}{2} i)] = \pi e^{-m}$$

(2)當 $m = 0$ 時, $f(z) = \frac{1}{1+z^2}$ 上半平面只有極點 i

$$\text{Res}[f(z); i] = \left. \frac{1}{2z} \right|_{z=i} = \frac{1}{2i} = -\frac{1}{2} i$$

$$\int_{-\infty}^{\infty} \frac{\cos mx dx}{1+x^2} = \text{Re}[2\pi i \cdot (-\frac{1}{2} i)] = \pi$$

(3)當 $m < 0$ 時, 上半平面只有極點 $-i$

$$\text{Res}[f(z); -i] = \left. \frac{e^{imz}}{2z} \right|_{z=-i} = \frac{e^{i(-mi)}}{-2i} = \frac{e^m}{2} i$$

$$\int_{-\infty}^{\infty} \frac{\cos mx dx}{1+x^2} = \text{Re}[-2\pi i \cdot (\frac{e^m}{2} i)] = \pi e^m$$

$$\int_{-\infty}^{\infty} \frac{\cos x dx}{1+x^2} = \pi e^{-|m|}$$

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