求過三點$P_1(1, -1, 2)$, $P_2(3, 0, 0)$, $P_3(4, 2, 1)$之平面方程式。[105高第一環安甲5(b)]

[解]$\overrightarrow{P_1P_2} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\overrightarrow{P_1P_3} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -2 \\ 3 & 3 & -1 \end{vmatrix} = (-\mathbf{i} + 6\mathbf{k} - 6\mathbf{j}) - (3\mathbf{k} - 6\mathbf{i} - 2\mathbf{j}) = 5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$$

平面法向量$\mathbf{n} = 5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$

設平面上任一點為$X(x, y, z)$，則

$$\mathbf{n} \cdot \overline{P_1X} = 0 \Rightarrow (5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) \cdot [(x-1)\mathbf{i} + (y+1)\mathbf{j} + (z-2)\mathbf{k}] = 0 \Rightarrow 5x - 4y + 3z = 15$$