已知 $f(x, y, z) = 2x - y^2 + z^2$，(1)求$\nabla f$；(2)求由坐標(4, –4, 2)處指向原點方向的單位向量 **u**；(3)求往上述 **u** 方向之 directional derivative $D_u f(4, -4, 2)$。[104 高應大機械甲丙 6]

[解](1)$\nabla f = \dfrac{\partial f}{\partial x}\mathbf{i} + \dfrac{\partial f}{\partial y}\mathbf{j} + \dfrac{\partial f}{\partial z}\mathbf{k} = 2\mathbf{i} - 2y\mathbf{j} + 2z\mathbf{k}$

(2)$\mathbf{u} = \dfrac{-4\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}}{\sqrt{(-4)^2 + 4^2 + (-2)^2}} = \dfrac{-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}}{3}$

(3)$\nabla f\big|_{(4, -4, 2)} = 2\mathbf{i} - 2(-4)\mathbf{j} + 2 \cdot 2\mathbf{k} = 2\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$

$D_u f(4, -4, 2) = \nabla f\big|_{(4, -4, 2)} \cdot \mathbf{u} = (2\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}) \cdot \dfrac{-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}}{3} = \dfrac{-4 + 16 - 4}{3} = \dfrac{8}{3}$